Technical Notes

Angle of Attack from Body-Fixed Rate Gyros

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Nomenclature

x, y, z = body fixed principal axes of inertia
$\mathbf{i}, \mathbf{j}, \mathbf{k}$ = unit vectors along x, y, z, respectively
p, q, r = rotational velocity components (roll, pitch, yaw) along x, y, z, respectively, rad/sec
A = reference area, ft²
C_m = aerodynamic pitching moment coefficient
d = reference length, ft
F = $\cot\alpha - k \csc\alpha$
$\mathbf{H}$ = angular momentum vector, ft-lb-sec
G = $-\int_0^\alpha C_m d\alpha$
I = moment of inertia about the y or z axis, slug-ft²
I_x = moment of inertia about the x axis, slug-ft²
k = angle-of-attack envelope parameter, $(1 + \eta_0^2)^{1/2} \cos\beta$
$\bar{q}$ = dynamic pressure, psf
t = time, sec
$\mathbf{u}$ = unit vector along the velocity vector
V = velocity, fps
α = total angle of attack
β = angle between $\mathbf{H}_0$ and $\mathbf{u}$
γ = $\arctan|r/q|$
δ = half cone angle of precession about $\mathbf{H}_0$ prior to atmospheric entry
$\Delta(\ )$ = $(\ )_2 - (\ )_1$, $\dot{\alpha} = \dot{\Omega} = 0$ at t_1 and t_2
ϵ = angle defining the plane of the angle of attack with respect to the body fixed axes
η = $I\Omega/I_x p$
ρ = atmospheric density, slugs/ft³
σ = $(I - I_x)p/I$, rad/sec
Ω = total pitching rate, $(q^2 + r^2)^{1/2}$, rad/sec

Subscripts

$(\ )_0$ = a quantity evaluated prior to atmospheric entry
$(\ )_{1,2}$ = quantities evaluated at successive times at which $\dot{\alpha} = \dot{\Omega} = 0$, $t_2 > t_1$

For a symmetrical body entering an atmosphere at a high velocity, a simple relation has been found between the envelope of the angle of attack and the body-referenced angular rates (roll, pitch, and yaw) as determined by body-fixed rate gyros. Attention had been brought to bear on gyros since accelerometers lacked the required sensitivity at very high altitudes where the dynamic pressure is low. The analysis assumes a constant roll rate, neglects lift forces, gravity, and aerodynamic damping but allows large angles of attack. It is independent of the weight and aerodynamic coefficients of the body, atmospheric density, altitude, velocity (except that it be large), Mach number and also applies to bodies that are aerodynamically unstable. The analysis exhibits good agreement with the results of a complete six degree of freedom digital calculation and has been used successfully to reduce actual flight data. An outgrowth of the investigation enables the atmospheric density to be ascertained if the velocity and the aerodynamic pitching moment coefficient characteristics are known.

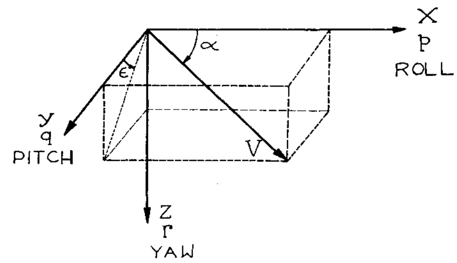

Fig. 1 Body-fixed triad.

Consider $d(\mathbf{H}\cdot\mathbf{u})/dt = \mathbf{H}\cdot(d\mathbf{u}/dt) + \mathbf{u}\cdot(d\mathbf{H}/dt)$. By the rotational equations of motion for a rigid body, $d\mathbf{H}/dt$ represents the aerodynamic moment and is perpendicular to the plane formed by the x axis and the velocity vector so that $\mathbf{u}\cdot(d\mathbf{H}/dt) = 0$. By the assumption of negligible lift forces and gravity, the direction of the velocity vector remains constant so that $d\mathbf{u}/dt = 0$; finally then, $\mathbf{H}\cdot\mathbf{u} = \text{const}$. Thus, an integral of the equations of motion is realized through the component of angular momentum along the velocity vector remaining a constant. From Fig. 1, $\mathbf{u} = \mathbf{i}\cos\alpha + \mathbf{j}\sin\alpha\cos\epsilon + \mathbf{k}\sin\alpha\sin\epsilon$, so

$$\mathbf{H}\cdot\mathbf{u} = I_x p\cos\alpha + I\Omega\sin\alpha\cos(\epsilon - \phi) = \text{const} \quad (1)$$

where $q = \Omega\cos\phi$ and $r = \Omega\sin\phi$.

Since $d\mathbf{u}/dt = 0$, it can be shown that $\mathbf{i}\cdot(d\mathbf{u}/dt) = 0$ implies

$$\dot{\alpha} = q\sin\epsilon - r\cos\epsilon = \Omega\sin(\epsilon - \phi) \quad (2)$$

remembering the kinematic relations for rigid body motion, $\mathbf{i}\cdot(d\mathbf{j}/dt) = -r$ and $\mathbf{i}\cdot(d\mathbf{k}/dt) = q$. If the rotational equations of motion,

$$I\dot{q} - (I - I_x)pr = C_m\bar{q}Ad\sin\epsilon \quad (3)$$

$$I\dot{r} + (I - I_x)pq = -C_m\bar{q}Ad\cos\epsilon \quad (4)$$

are substituted into Eq. (2), there results

$$\dot{\alpha} = I\Omega\dot{\Omega}/C_m\bar{q}Ad \quad (5)$$

Thus, $\dot{\alpha} = 0$ when $\dot{\Omega} = 0$, i.e., the extrema of the angle of attack can be located with respect to time by zero slopes of the time history curve of the total pitching rate, Ω. Further-

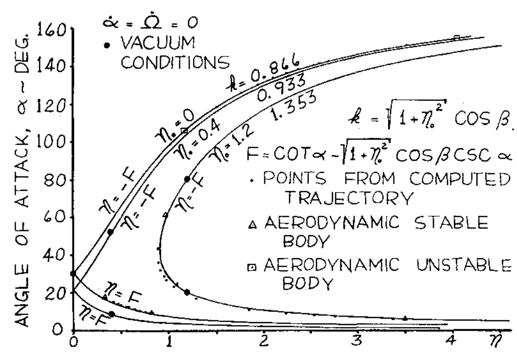

Fig. 2 Angle-of-attack extremum curves for $\beta = 30°$.

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more, if $\dot{\alpha} = 0$, then $\epsilon - \phi = 0$ or π so that $\cos(\epsilon - \phi) = \pm 1$ in Eq. (1). Finally then,

$$\pm I\Omega/I_x p = \cot\alpha - k \csc\alpha \quad \text{at} \quad \dot{\alpha} = \dot{\Omega} = 0$$

that is, $\eta = \pm F(\alpha, k)$ where k is some constant.

It can be seen that $k = \mathbf{H}\cdot\mathbf{u}/I_x p$ ($p \neq 0$), and, if evaluated prior to entry into an atmosphere, $k = (1 + \eta_0^2)^{1/2} \cos\beta = \cos\beta/\cos\delta$ where β represents the time average angle of attack and δ represents the precessional half cone angle at the high altitudes above the sensible atmosphere. It might be noted from Eq. (5) that, since $C_m < 0$ for an aerodynamically stable body, $\min\Omega$ corresponds to $\max\alpha$ and vice versa. The analysis shows good agreement with six degree of freedom digital program results as shown by Fig. 2. The curves $\eta = +F(\alpha, k)$ associated with large angles of attack are usually the regions of interest.

Though η and thus η_0 are known from body-fixed rate gyro flight data, as a rule, only an estimate of β can be made from a trajectory analysis, thus necessitating a more precise determination of k. To ascertain k for $\eta = +F$, invert this relation to give $(1 + \eta^2) \sin\alpha = -\eta k + \xi$ and $(1 + \eta^2) \cos\alpha = k + \eta\xi$ where $\xi = (1 + \eta^2 - k^2)^{1/2}$.

Consider successive times at which $\dot{\alpha} = \dot{\Omega} = 0$, then

$$(1 + \eta_1^2)(1 + \eta_2^2) \sin\Delta\alpha = k(\xi_2 - \xi_1)(1 + \eta_1\eta_2) - (k^2 + \xi_1\xi_2)(\eta_2 - \eta_1)$$

$$(1 + \eta_1^2)(1 + \eta_2^2) \cos\Delta\alpha = k(\xi_2 - \xi_1)(\eta_2 - \eta_1) + (k^2 + \xi_1\xi_2)(1 + \eta_1\eta_2)$$

Eliminating $k(\xi_2 - \xi_1)$ gives

$$\xi_1\xi_2 = (1 + \eta_1\eta_2) \cos\Delta\alpha - (\eta_2 - \eta_1) \sin\Delta\alpha - k^2$$

Squaring both sides eliminates k^4, and then it can be shown that $k = k^* \sin(\theta - |\Delta\alpha|)$ where

$$k^* = [(1 + \eta_1^2)^{-1} + (1 + \eta_2^2)^{-1} - 2(1 + \eta_1^2)^{-1/2}(1 + \eta_2^2)^{-1/2} \cos(\theta - |\Delta\alpha|)]^{-1/2}$$

and

$$\theta = |\arctan\eta_2 - \arctan\eta_1|$$

From Eqs. (2–4), it can be shown that

$$\Delta\alpha = -\int_{t_1}^{t_2} (q\dot{q} + r\dot{r})[(\dot{q} - \sigma r)^2 + (\dot{r} + \sigma q)^2]^{-1/2} dt$$

An evaluation of the envelope parameter k several times (i.e., for several pair of points where $\dot{\alpha} = \dot{\Omega} = 0$) serves as a check on the data reduction program since k should be a consant for any given flight.

In order to avoid evaluating $\dot{q}$ and $\dot{r}$, the following approximation is proposed:

$$\Delta\alpha \cong -\frac{\Delta\eta}{\lambda - (I/I_x) + 1} \qquad \lambda > I/I_x$$

where

$$\lambda = I(N\pi - \Delta\epsilon)/I_x p\Delta t \qquad N = 0, \pm 1, \pm 2 \ldots$$

and ϵ is determined from Table 1 wherein $\gamma = \arctan|r/q|$, $0 \leq \gamma \leq \pi/2$.

The integer N is chosen on the basis that the computed $\Delta\alpha$ will give a value of k which matches the estimated value given by $k = (1 + \eta_0^2)^{1/2} \cos\beta$ and a trajectory analysis.

The analysis for the case where there is no spin ($p = 0$) is similar and simpler:

$$\Omega \sin\alpha = [1 + (\Omega_2/\Omega_1)^2 - 2(\Omega_2/\Omega_1) \cos\Delta\alpha]^{-1/2}\Omega_2 \sin|\Delta\alpha|$$

When the angle-of-attack oscillations are fairly well developed, an approximate value of density can be found by integrating Eq. (5), thus

Table 1 Plane of angle of attack

q	r	ϵ
+	+	$\pi + \gamma$
+	−	$\pi - \gamma$
−	+	$2\pi - \gamma$
−	−	γ

$$\rho_m \cong -I\Delta\Omega^2/AdV^2\Delta G$$

where

$$G = -\int_0^\alpha C_m d\alpha$$

and

$$t_m = (t_1 + t_2)/2$$

Thus, the additional knowledge of velocity, the aerodynamic pitching moment coefficient, and, presumably, altitude is needed.

An Experimental Evaluation of Plug Cluster Nozzles

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Analysis

If we define the clustered nozzle exit area ratio (ϵ_c) as the circular area enclosing all units divided by the total throat area, and the unit nozzle area ratio (ϵ_N) in the conventional manner, then, for nozzles touching,

$$\epsilon_c/\epsilon_N = (\pi/N + \cos\theta)^2/N$$

where N is the number of units, and θ is the unit nozzle inclination angle.

This simple geometric relationship, combined with well-known isentropic relationships connecting the Prandtl-Meyer angle to the increase in area ratio (from ϵ_N to ϵ_c), is quite helpful in indicating the practical range for a clustered arrangement. For example, a typical rocket motor with $\epsilon_c = 40$ and $\epsilon_N = 7.5$ requires 44 units and a 27° inclination angle if axial flow at ϵ_c is desired. Also, the cluster arrangement is found to be mainly attractive with more than 12 units. With a lower number of units, only a small difference between ϵ_N and ϵ_c is obtained, the individual nozzles are directed almost axially, and the total thrust is essentially the sum of the individual nozzle thrusts.

Test Model and Facilities

All of the data were obtained with high-pressure cold dry air. The plug contours were designed by the axisymmetric method of characteristics assuming annular flow. The individual units were truncated perfect nozzles.[1] The 24-unit model, shown schematically in Fig. 1, was designed with $\epsilon_N = 4$ and $\epsilon_c = 16$. Axial and lateral forces were measured directly

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